

COMPUTATIONAL DATA ON THE DECOMPOSITION NUMBERS OF $\mathrm{SO}_8^+(2^f)$

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This file contains the details of the computations related to Sections 4 and 5 of the work *On the decomposition numbers of $\mathrm{SO}_8^+(2^f)$* of the author; we keep in this file the notation introduced there. The data are collected in Tables 1 and 2 respectively.

We collect in Table 1 all values of the family sums $\mathcal{S}(\varphi, \mathcal{C})$ as in Section 4 of the paper. The first column labels each family \mathcal{C} of conjugacy class representatives of U , while the other columns contain the values of the irreducible characters of U taken into consideration in Section 3 of the work.

Family sum	$\psi_{\mathrm{lin}}^{0,0,0,0}$	$\psi_{\mathrm{lin}}^{1,0,0,0}$	$\psi_{\mathrm{lin}}^{1,1,0,0}$	$\psi_{8,9,10,q^3/2}^{s,t,1,1,1,1}$	$\psi_{\mathrm{lin}}^{1,1,1,0}$	$\psi_{5,6,7}^{1,1,1,0,1,d}$	$\psi_{\mathrm{lin}}^{1,1,1,1}$
Trivial	1	1	1	$q^3/2$	1	q	1
$\mathcal{S}(\cdot, \mathcal{C}_1^2)$	$q-1$	-1	-1	0	-1	0	-1
$\mathcal{S}(\cdot, \mathcal{C}_2^2)$	$q-1$	$q-1$	-1	0	-1	0	-1
$\mathcal{S}(\cdot, \mathcal{C}_3^2)$	$q-1$	$q-1$	$q-1$	$\phi(t)q^2/2$	-1	0	-1
$\mathcal{S}(\cdot, \mathcal{C}_4^2)$	$q-1$	$q-1$	$q-1$	0	$q-1$	0	-1
$\mathcal{S}(\cdot, \mathcal{C}_5^2)$	$q-1$	$q-1$	$q-1$	0	$q-1$	$-q$	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_6^2)$	$q-1$	$q-1$	$q-1$	0	$q-1$	$-q$	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_7^2)$	$q-1$	$q-1$	$q-1$	0	$q-1$	$-q$	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_8^2)$	$q-1$	$q-1$	$q-1$	$-q^3/2$	$q-1$	$q(q-1)$	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_9^2)$	$q-1$	$q-1$	$q-1$	$-q^3/2$	$q-1$	$q(q-1)$	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_{10}^2)$	$q-1$	$q-1$	$q-1$	$-q^3/2$	$q-1$	$q(q-1)$	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_{11}^2)$	$q-1$	$q-1$	$q-1$	$q^3(q-1)/2$	$q-1$	$q(q-1)$	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_{12}^2)$	$q-1$	$q-1$	$q-1$	$q^3(q-1)/2$	$q-1$	$q(q-1)$	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,q^8}^3)$	$(q-1)^2$	$-(q-1)$	1	0	1	$-q$	1
$\mathcal{S}(\cdot, \mathcal{C}_{4,q^8}^3)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	0	$(q-1)^2$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8}^3)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	$-\phi(t)q^2/2$	$-(q-1)$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{5,6,q^9}^3)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	0	$(q-1)^2$	q	$(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{7,q^9}^3)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	0	$(q-1)^2$	$-q(q-1)$	$(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{8,9,10,q^{10}}^3)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	$q^3/2$	$(q-1)^2$	$q(q-1)^2$	$(q-1)^2$

Family sum	$\psi_{\text{lin}}^{0,0,0,0}$	$\psi_{\text{lin}}^{1,0,0,0}$	$\psi_{\text{lin}}^{1,1,0,0}$	$\psi_{8,9,10,q^3/2}^{s,t,1,1,1,1}$	$\psi_{\text{lin}}^{1,1,1,0}$	$\psi_{5,6,7}^{1,1,1,0,1,d}$	$\psi_{\text{lin}}^{1,1,1,1}$
$\mathcal{S}(\cdot, \mathcal{C}_{1,4,q^8}^4)$	$(q-1)^2$	$-(q-1)$	$-(q-1)$	0	$-(q-1)$	$-q$	1
$\mathcal{S}(\cdot, \mathcal{C}_{2,q^8}^4)$	$(q-1)^2$	$(q-1)^2$	$-(q-1)$	0	$-(q-1)$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8}^4)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	$-\phi(t)q^2/2$	$-(q-1)$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{5,7,q^9}^4)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	0	$(q-1)^2$	q	$(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{6,q^9}^4)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	0	$(q-1)^2$	$-q(q-1)$	$(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{8,9,10,q^{10}}^4)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	$q^3/2$	$(q-1)^2$	$q(q-1)^2$	$(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{2,4,q^8}^5)$	$(q-1)^2$	$(q-1)^2$	$-(q-1)$	0	$-(q-1)$	$-q$	1
$\mathcal{S}(\cdot, \mathcal{C}_{1,q^8}^5)$	$(q-1)^2$	$-(q-1)$	$-(q-1)$	0	$-(q-1)$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8}^5)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	$-\phi(t)q^2/2$	$-(q-1)$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{6,7,q^9}^5)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	0	$(q-1)^2$	q	$(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{5,q^9}^5)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	0	$(q-1)^2$	$-q(q-1)$	$(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{8,9,10,q^{10}}^5)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	$q^3/2$	$(q-1)^2$	$q(q-1)^2$	$(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,4,q^7}^{6,p=2})$	$(q-1)^3$	$-(q-1)^2$	$q-1$	$\phi(s)q^3/2$	$q-1$	$2q$	-1
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,q^8}^6)$	$(q-1)^3$	$-(q-1)^2$	$q-1$	0	$q-1$	$-q(q-1)$	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_{1,4,q^8}^6)$	$(q-1)^3$	$-(q-1)^2$	$-(q-1)^2$	0	$-(q-1)^2$	$-q(q-1)$	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_{2,4,q^8}^6)$	$(q-1)^3$	$(q-1)^3$	$-(q-1)^2$	0	$-(q-1)^2$	$-q(q-1)$	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_{5,6,7,q^8}^6)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	$-q^3/2$	$(q-1)^3$	$-q$	$(q-1)^3$
$\mathcal{S}(\cdot, \mathcal{C}_{5,6,q^9}^6)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	0	$(q-1)^3$	$q(q-1)$	$(q-1)^3$
$\mathcal{S}(\cdot, \mathcal{C}_{5,7,q^9}^6)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	0	$(q-1)^3$	$q(q-1)$	$(q-1)^3$
$\mathcal{S}(\cdot, \mathcal{C}_{6,7,q^9}^6)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	0	$(q-1)^3$	$q(q-1)$	$(q-1)^3$
$\mathcal{S}(\cdot, \mathcal{C}_{8,9,10,q^{10}}^6)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	$-q^3/2$	$(q-1)^3$	$q(q-1)^3$	$(q-1)^3$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8}^6)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	$\phi(t)q^2/2$	$-(q-1)^2$	0	$-(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8,1}^6)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	$\phi(t)q^2/2$	$-(q-1)^2$	0	$-(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8,2}^6)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	$\phi(t)q^2/2$	$-(q-1)^2$	0	$-(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8,3}^6)$	$(q-1)^4$	$(q-1)^4$	$(q-1)^4$	$-\phi(t)q^2/2$	$-(q-1)^3$	0	$-(q-1)^3$
$\mathcal{S}(\cdot, \mathcal{C}_{1,3,q^5}^7)$	$(q-1)^2$	$-(q-1)$	$-(q-1)$	0	1	0	1
$\mathcal{S}(\cdot, \mathcal{C}_{2,3,q^5}^7)$	$(q-1)^2$	$(q-1)^2$	$-(q-1)$	0	1	0	1
$\mathcal{S}(\cdot, \mathcal{C}_{3,4,q^5}^7)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	0	$-(q-1)$	0	1
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,4,2q^7}^{7,p=2})$	$(q-1)^4$	$-(q-1)^3$	$(q-1)^2$	$-\phi(s)q^3/2$	$(q-1)^2$	$2q(q-1)$	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,q^6}^7)$	$(q-1)^3$	$-(q-1)^2$	$q-1$	0	$(q-1)$	q	$(q-1)$

Family sum	$\psi_{\text{lin}}^{0,0,0,0}$	$\psi_{\text{lin}}^{1,0,0,0}$	$\psi_{\text{lin}}^{1,1,0,0}$	$\psi_{8,9,10,q^3/2}^{s,t,1,1,1,1}$	$\psi_{\text{lin}}^{1,1,1,0}$	$\psi_{5,6,7}^{1,1,1,0,1,d}$	$\psi_{\text{lin}}^{1,1,1,1}$
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,q^7}^7)$	$(q-1)^3$	$-(q-1)^2$	$q-1$	0	$(q-1)$	$-q(q-1)$	$(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{1,4,q^6}^7)$	$(q-1)^3$	$-(q-1)^2$	$-(q-1)^2$	0	$-(q-1)^2$	q	$(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{1,4,q^7}^7)$	$(q-1)^3$	$-(q-1)^2$	$-(q-1)^2$	0	$-(q-1)^2$	$-q(q-1)$	$(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{2,4,q^6}^7)$	$(q-1)^3$	$(q-1)^3$	$-(q-1)^2$	0	$-(q-1)^2$	q	$(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{2,4,q^7}^7)$	$(q-1)^3$	$(q-1)^3$	$-(q-1)^2$	0	$-(q-1)^2$	$-q(q-1)$	$(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{1,q^6,1}^7)$	$(q-1)^2$	$-(q-1)$	$-(q-1)$	0	$-(q-1)$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{1,q^6,2}^7)$	$(q-1)^2$	$-(q-1)$	$-(q-1)$	0	$-(q-1)$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{1,q^7}^7)$	$(q-1)^2$	$-(q-1)$	$-(q-1)$	0	$-(q-1)$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{2,q^6,1}^7)$	$(q-1)^2$	$(q-1)^2$	$-(q-1)$	0	$-(q-1)$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{2,q^6,2}^7)$	$(q-1)^2$	$(q-1)^2$	$-(q-1)$	0	$-(q-1)$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{2,q^7}^7)$	$(q-1)^2$	$(q-1)^2$	$-(q-1)$	0	$-(q-1)$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{4,q^6,1}^7)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	0	$(q-1)^2$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{4,q^6,2}^7)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	0	$(q-1)^2$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{4,q^7}^7)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	0	$(q-1)^2$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8,1}^7)$	$\frac{(q-1)^4(q-2)}{2}$	$\frac{(q-1)^4(q-2)}{2}$	$\frac{(q-1)^4(q-2)}{2}$	$\frac{-\phi(t)q^2(q-2)}{4}$	$-\frac{(q-1)^3(q-2)}{2}$	0	$\frac{(q-1)^3(q-2)}{2}$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8,2}^7)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	$\frac{\phi(t)q^2(q-1)}{2}$	$-(q-1)$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8,3}^7)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	$-\frac{\phi(t)q^2(q-1)}{2}$	$-(q-1)^2$	0	$-(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8,4}^7)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	$-\frac{\phi(t)q^2(q-1)}{2}$	$-(q-1)^2$	0	$-(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8,5}^7)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	$-\frac{\phi(t)q^2(q-1)}{2}$	$-(q-1)^2$	0	$-(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8,6}^7)$	$(q-1)^4$	$(q-1)^4$	$(q-1)^4$	$\frac{\phi(t)q^2(q-1)}{2}$	$-(q-1)^3$	0	$-(q-1)^3$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8,7}^7)$	$(q-1)^4$	$(q-1)^4$	$(q-1)^4$	$\frac{\phi(t)q^2(q-1)}{2}$	$-(q-1)^3$	0	$-(q-1)^3$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8,8}^7)$	$(q-1)^4$	$(q-1)^4$	$(q-1)^4$	$\frac{\phi(t)q^2(q-1)}{2}$	$-(q-1)^3$	0	$-(q-1)^3$
$\mathcal{S}(\cdot, \mathcal{C}_{5,6,7,2q^8}^7)$	$(q-1)^4$	$(q-1)^4$	$(q-1)^4$	$q^3/2$	$(q-1)^4$	$-q(q-1)$	$(q-1)^4$
$\mathcal{S}(\cdot, \mathcal{C}_{5,6,q^8}^7)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	0	$(q-1)^3$	$q(q-1)$	$(q-1)^3$
$\mathcal{S}(\cdot, \mathcal{C}_{5,7,q^8}^7)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	0	$(q-1)^3$	$q(q-1)$	$(q-1)^3$
$\mathcal{S}(\cdot, \mathcal{C}_{6,7,q^8}^7)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	0	$(q-1)^3$	$q(q-1)$	$(q-1)^3$
$\mathcal{S}(\cdot, \mathcal{C}_{5,q^8}^7)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	0	$(q-1)^2$	$-q(q-1)$	$(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{6,q^8}^7)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	0	$(q-1)^2$	$-q(q-1)$	$(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{7,q^8}^7)$	$(q-1)^2$	$(q-1)^2$	$(q-1)^2$	0	$(q-1)^2$	$-q(q-1)$	$(q-1)^2$

Family sum	$\psi_{\text{lin}}^{0,0,0,0}$	$\psi_{\text{lin}}^{1,0,0,0}$	$\psi_{\text{lin}}^{1,1,0,0}$	$\psi_{8,9,10,q^3/2}^{s,t,1,1,1,1}$	$\psi_{\text{lin}}^{1,1,1,0}$	$\psi_{5,6,7}^{1,1,1,0,1,d}$	$\psi_{\text{lin}}^{1,1,1,1}$
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,4,2q^7}^{8,p=2})$	$(q-1)^4$	$-(q-1)^3$	$(q-1)^2$	$\frac{-\phi(s)q^3}{2}$	$(q-1)^2$	$2q(q-1)$	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{3,q^8}^8)$	$\frac{q(q-1)^4}{2}$	$\frac{q(q-1)^4}{2}$	$\frac{q(q-1)^4}{2}$	$\frac{-\phi(t)q^3}{4}$	$\frac{-q(q-1)^3}{2}$	0	$\frac{-q(q-1)^3}{2}$
$\mathcal{S}(\cdot, \mathcal{C}_{5,6,7,2q^8}^8)$	$(q-1)^4$	$(q-1)^4$	$(q-1)^4$	$q^3/2$	$(q-1)^4$	$-q(q-1)$	$(q-1)^4$
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,3,q^5}^9)$	$(q-1)^3$	$-(q-1)^2$	$q-1$	0	-1	0	-1
$\mathcal{S}(\cdot, \mathcal{C}_{3,4,q^5}^9)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	0	$-(q-1)^2$	0	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,4,q^6}^{9,p=2})$	$(q-1)^4$	$-(q-1)^3$	$(q-1)^2$	0	$(q-1)^2$	$-2q$	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,q^6}^9)$	$(q-1)^3$	$-(q-1)^2$	$q-1$	0	$q-1$	q	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_{4,q^6}^9)$	$(q-1)^3$	$(q-1)^3$	$(q-1)^3$	0	$(q-1)^3$	0	$-(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{1,3,4,q^5}^{10})$	$(q-1)^3$	$-(q-1)^2$	$-(q-1)^2$	0	$q-1$	0	-1
$\mathcal{S}(\cdot, \mathcal{C}_{2,3,q^5}^{10})$	$(q-1)^3$	$(q-1)^3$	$-(q-1)^2$	0	$q-1$	0	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,4,q^6}^{10,p=2})$	$(q-1)^4$	$-(q-1)^3$	$(q-1)^2$	0	$(q-1)^2$	$-2q$	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{1,4,q^6}^{10})$	$(q-1)^3$	$-(q-1)^2$	$-(q-1)^2$	0	$-(q-1)^2$	q	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_{2,q^6}^{10})$	$(q-1)^3$	$(q-1)^3$	$-(q-1)^2$	0	$-(q-1)^2$	0	$-(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{2,3,4,q^5}^{11})$	$(q-1)^3$	$(q-1)^3$	$-(q-1)^2$	0	$q-1$	0	-1
$\mathcal{S}(\cdot, \mathcal{C}_{1,3,q^5}^{11})$	$(q-1)^3$	$-(q-1)^2$	$-(q-1)^2$	0	$q-1$	0	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,4,q^6}^{11,p=2})$	$(q-1)^4$	$-(q-1)^3$	$(q-1)^2$	0	$(q-1)^2$	$-2q$	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{2,4,q^6}^{11})$	$(q-1)^3$	$(q-1)^3$	$-(q-1)^2$	0	$-(q-1)^2$	q	$q-1$
$\mathcal{S}(\cdot, \mathcal{C}_{1,q^6}^{11})$	$(q-1)^3$	$-(q-1)^2$	$-(q-1)^2$	0	$-(q-1)^2$	0	$-(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,3,q^5}^{12})$	$(q-1)^4$	$-(q-1)^3$	$(q-1)^2$	0	$-(q-1)$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{1,3,4,q^5}^{12})$	$(q-1)^4$	$-(q-1)^3$	$-(q-1)^3$	0	$(q-1)^2$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{2,3,4,q^5}^{12})$	$(q-1)^4$	$(q-1)^4$	$-(q-1)^3$	0	$(q-1)^2$	0	$-(q-1)$
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,4,q^6}^{12,p=2})$	$(q-1)^4(q-2)$	$-(q-1)^3(q-2)$	$(q-1)^2(q-2)$	0	$(q-1)^2(q-2)$	$4q$	$-(q-1)(q-2)$
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,q^6}^{12})$	$(q-1)^4$	$-(q-1)^3$	$(q-1)^2$	0	$(q-1)^2$	$-q$	$(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{1,4,q^6}^{12})$	$(q-1)^4$	$-(q-1)^3$	$-(q-1)^3$	0	$-(q-1)^3$	$-q$	$(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{2,4,q^6}^{12})$	$(q-1)^4$	$(q-1)^4$	$-(q-1)^3$	0	$-(q-1)^3$	$-q$	$(q-1)^2$
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,3,4}^{13})$	$(q-1)^4$	$-(q-1)^3$	$(q-1)^2$	$\phi(s+t)q^2/2$	$-(q-1)$	0	1
$\mathcal{S}(\cdot, \mathcal{C}_{1,2,3,4}^{14})$	$(q-1)^4$	$-(q-1)^3$	$(q-1)^2$	$-\phi(s+t)q^2/2$	$-(q-1)$	0	1

TABLE 1. The family sums of the induction of the irreducible characters of U to G .

Table 2 contains for each $w \in W$ the decompositions of the Deligne-Lusztig characters R_w with respect to the ℓ -projective characters Ψ_1, \dots, Ψ_{14} labelling the columns of the ℓ -decomposition matrix of G when $\ell \mid q + 1$, that is, the coefficients $a_{w,i}$ for every $w \in W$ and every $i \in \{1, \dots, 14\}$ as in §5.7 of the paper. We denote by $[i_1, \dots, i_m]$ the element $s_{i_1} \cdots s_{i_m}$ of W .

Element $w \in W$	Decomposition of R_w
$[\]$	$+\Psi_1 + 2\Psi_2 + 8\Psi_7$
$[4]$	$+\Psi_1 - 2\Psi_8$
$[2]$	$+\Psi_1 - 2\Psi_8$
$[3]$	$+\Psi_1 - 2\Psi_8$
$[1]$	$+\Psi_1 - 2\Psi_8$
$[2, 4]$	$+\Psi_1 - 2\Psi_2 + 4\Psi_3 - 4\Psi_8 + 4\Psi_{10}$
$[3, 1]$	$+\Psi_1 - \Psi_2 - \Psi_7$
$[4, 3]$	$+\Psi_1 - 2\Psi_2 + 4\Psi_4 - 4\Psi_8 + 4\Psi_{11}$
$[2, 3]$	$+\Psi_1 - \Psi_2 - \Psi_7$
$[3, 2]$	$+\Psi_1 - 2\Psi_2 + 4\Psi_5 - 4\Psi_8 + 4\Psi_{12}$
$[3, 4]$	$+\Psi_1 - \Psi_2 - \Psi_7$
$[1, 4]$	$+\Psi_1 - \Psi_2 - \Psi_7$
$[1, 2]$	$+\Psi_1 - \Psi_2 - \Psi_7$
$[1, 3]$	$+\Psi_1 - \Psi_2 - \Psi_7$
$[4, 3, 1]$	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
$[2, 3, 1]$	$+\Psi_1 - 4\Psi_2 + 4\Psi_3 + 4\Psi_4 + 4\Psi_5 - 8\Psi_6 - 6\Psi_8 + 4\Psi_{10} + 4\Psi_{11} + 4\Psi_{12}$
$[2, 4, 3]$	$+\Psi_1 - 2\Psi_8$
$[3, 1, 2]$	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
$[3, 1, 4]$	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
$[4, 3, 2]$	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
$[2, 3, 2]$	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
$[3, 4, 3]$	$+\Psi_1 - 2\Psi_8$
$[2, 3, 4]$	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
$[3, 2, 4]$	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
$[1, 2, 4]$	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
$[1, 3, 1]$	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
$[1, 4, 3]$	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
$[1, 2, 3]$	$+\Psi_1 - 2\Psi_8$
$[1, 3, 2]$	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
$[1, 3, 4]$	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
$[3, 1, 2, 3]$	$+\Psi_1 - \Psi_2 - \Psi_7$
$[2, 4, 3, 1]$	$+\Psi_1 - \Psi_2 - \Psi_7$
$[3, 1, 4, 3]$	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
$[4, 3, 1, 2]$	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$

Element $w \in W$	Decomposition of R_w
[2, 3, 1, 2]	$+\Psi_1 - \Psi_2 - \Psi_7$
[3, 4, 3, 1]	$+\Psi_1 - \Psi_2 - \Psi_7$
[2, 3, 1, 4]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[2, 4, 3, 2]	$+\Psi_1 - \Psi_2 - \Psi_7$
[2, 3, 4, 3]	$+\Psi_1 - \Psi_2 - \Psi_7$
[3, 1, 2, 4]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[3, 4, 3, 2]	$+\Psi_1 - \Psi_2 - \Psi_7$
[2, 3, 2, 4]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[3, 2, 4, 3]	$+\Psi_1 - \Psi_2 - \Psi_7$
[1, 4, 3, 1]	$+\Psi_1 - \Psi_2 - \Psi_7$
[1, 2, 3, 1]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[1, 2, 4, 3]	$+\Psi_1 - \Psi_2 - \Psi_7$
[1, 3, 1, 2]	$+\Psi_1 - 2\Psi_2 + 4\Psi_3 - 4\Psi_8 + 4\Psi_{10}$
[1, 3, 1, 4]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[1, 4, 3, 2]	$+\Psi_1 - 2\Psi_2 + 4\Psi_4 - 4\Psi_8 + 4\Psi_{11}$
[1, 2, 3, 2]	$+\Psi_1 - 2\Psi_2 + 4\Psi_5 - 4\Psi_8 + 4\Psi_{12}$
[1, 3, 4, 3]	$+\Psi_1 - \Psi_2 - \Psi_7$
[1, 2, 3, 4]	$+\Psi_1 - \Psi_2 - \Psi_7$
[1, 3, 2, 4]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[4, 3, 1, 2, 3]	$+\Psi_1 - 2\Psi_8$
[3, 1, 2, 3, 4]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[2, 3, 1, 2, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[3, 1, 4, 3, 1]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[3, 1, 4, 3, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[2, 3, 1, 4, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[2, 4, 3, 1, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[2, 3, 4, 3, 1]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[3, 4, 3, 1, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[3, 1, 2, 4, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[2, 3, 1, 2, 4]	$+\Psi_1 - 2\Psi_8$
[2, 3, 4, 3, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[3, 2, 4, 3, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[3, 2, 4, 3, 1]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[2, 3, 2, 4, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[1, 3, 1, 2, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[1, 2, 4, 3, 1]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[1, 3, 1, 4, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[1, 4, 3, 1, 2]	$+\Psi_1 - 2\Psi_8$
[1, 2, 3, 1, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[1, 3, 4, 3, 1]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[1, 2, 3, 1, 4]	$+\Psi_1 - 4\Psi_2 + 4\Psi_3 + 4\Psi_4 + 4\Psi_5 - 8\Psi_6 - 6\Psi_8 + 4\Psi_{10} + 4\Psi_{11} + 4\Psi_{12}$

Element $w \in W$	Decomposition of R_w
[3, 1, 4, 3, 1, 2, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[2, 3, 1, 2, 4, 3, 2]	$+\Psi_1 - 2\Psi_8$
[3, 1, 2, 3, 4, 3, 1]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[3, 1, 2, 4, 3, 1, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[2, 3, 4, 3, 1, 2, 3]	$+\Psi_1 - 4\Psi_2 + 4\Psi_3 + 4\Psi_4 + 4\Psi_5 - 8\Psi_6 - 6\Psi_8 + 4\Psi_{10} + 4\Psi_{11} + 4\Psi_{12}$
[3, 2, 4, 3, 1, 2, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[2, 3, 1, 2, 4, 3, 1]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[2, 3, 2, 4, 3, 1, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[1, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[1, 2, 4, 3, 1, 2, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[1, 2, 3, 1, 2, 3, 4]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[1, 3, 1, 4, 3, 1, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[1, 2, 3, 1, 4, 3, 2]	$+\Psi_1 - 4\Psi_2 + 4\Psi_3 + 4\Psi_4 + 4\Psi_5 - 8\Psi_6 - 6\Psi_8 + 4\Psi_{10} + 4\Psi_{11} + 4\Psi_{12}$
[1, 2, 3, 1, 4, 3, 1]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[1, 3, 1, 2, 3, 4, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[1, 3, 1, 2, 4, 3, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[1, 3, 4, 3, 1, 2, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[1, 2, 3, 1, 2, 4, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[1, 2, 3, 4, 3, 1, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[1, 3, 1, 2, 4, 3, 1]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[1, 3, 2, 4, 3, 1, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[1, 2, 3, 2, 4, 3, 1]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[1, 2, 3, 2, 4, 3, 2]	$+\Psi_1 - 4\Psi_2 + 4\Psi_3 + 4\Psi_4 + 4\Psi_5 - 8\Psi_6 - 6\Psi_8 + 4\Psi_{10} + 4\Psi_{11} + 4\Psi_{12}$
[3, 1, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - \Psi_2 - \Psi_7$
[2, 3, 1, 2, 3, 4, 3, 2]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[3, 1, 2, 3, 4, 3, 1, 2]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[2, 3, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 2\Psi_2 + 4\Psi_4 - 4\Psi_8 + 4\Psi_{11}$
[2, 3, 1, 4, 3, 1, 2, 3]	$+\Psi_1 - 2\Psi_2 + 4\Psi_5 - 4\Psi_8 + 4\Psi_{12}$
[3, 2, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - \Psi_2 - \Psi_7$
[2, 3, 1, 2, 3, 4, 3, 1]	$+\Psi_1 - 2\Psi_2 + 4\Psi_3 - 4\Psi_8 + 4\Psi_{10}$
[3, 1, 2, 4, 3, 1, 2, 3]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[2, 3, 1, 2, 4, 3, 1, 2]	$+\Psi_1 - \Psi_2 - \Psi_7$
[2, 3, 2, 4, 3, 1, 2, 3]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[1, 2, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - \Psi_2 - \Psi_7$
[1, 3, 1, 2, 3, 4, 3, 2]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[1, 2, 3, 1, 4, 3, 1, 2]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[1, 3, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - \Psi_2 - \Psi_7$
[1, 2, 3, 1, 2, 3, 4, 3]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[1, 3, 1, 4, 3, 1, 2, 3]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[1, 2, 3, 1, 2, 4, 3, 2]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[1, 3, 1, 2, 3, 4, 3, 1]	$+\Psi_1 - \Psi_2 - \Psi_7$

Element $w \in W$	Decomposition of R_w
[1, 3, 1, 2, 4, 3, 1, 2]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[1, 2, 3, 4, 3, 1, 2, 3]	$+\Psi_1 - \Psi_2 - \Psi_7$
[1, 3, 2, 4, 3, 1, 2, 3]	$+\Psi_1 - \Psi_2 - \Psi_7$
[1, 2, 3, 1, 2, 4, 3, 1]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[1, 2, 3, 2, 4, 3, 1, 2]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[3, 1, 2, 3, 4, 3, 1, 2, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[2, 3, 1, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[3, 1, 2, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 4\Psi_2 + 4\Psi_3 + 4\Psi_4 + 4\Psi_5 - 8\Psi_6 - 6\Psi_8 + 4\Psi_{10} + 4\Psi_{11} + 4\Psi_{12}$
[2, 3, 1, 2, 3, 4, 3, 1, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[2, 3, 2, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[2, 3, 1, 2, 4, 3, 1, 2, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[1, 3, 1, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[1, 2, 3, 1, 2, 3, 4, 3, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_5 - 2\Psi_{12}$
[1, 3, 1, 2, 3, 4, 3, 1, 2]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[1, 2, 3, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 4\Psi_2 + 4\Psi_3 + 4\Psi_4 + 4\Psi_5 - 8\Psi_6 - 6\Psi_8 + 4\Psi_{10} + 4\Psi_{11} + 4\Psi_{12}$
[1, 2, 3, 1, 4, 3, 1, 2, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[1, 3, 2, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 2\Psi_2 + 2\Psi_4 - 2\Psi_{11}$
[1, 2, 3, 1, 2, 3, 4, 3, 1]	$+\Psi_1 - 4\Psi_2 + 4\Psi_3 + 4\Psi_4 + 4\Psi_5 - 8\Psi_6 - 6\Psi_8 + 4\Psi_{10} + 4\Psi_{11} + 4\Psi_{12}$
[1, 3, 1, 2, 4, 3, 1, 2, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[1, 2, 3, 1, 2, 4, 3, 1, 2]	$+\Psi_1 - 2\Psi_8$
[1, 2, 3, 2, 4, 3, 1, 2, 3]	$+\Psi_1 - 2\Psi_2 + 2\Psi_3 - 2\Psi_{10}$
[3, 1, 2, 3, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[2, 3, 1, 2, 3, 4, 3, 1, 2, 3]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[2, 3, 1, 2, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[1, 3, 1, 2, 3, 4, 3, 1, 2, 3]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[1, 2, 3, 1, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 2\Psi_2 + 4\Psi_5 - 4\Psi_8 + 4\Psi_{12}$
[1, 3, 1, 2, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[1, 2, 3, 1, 2, 3, 4, 3, 1, 2]	$+\Psi_1 - 2\Psi_2 + 4\Psi_4 - 4\Psi_8 + 4\Psi_{11}$
[1, 2, 3, 2, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 3\Psi_2 + 2\Psi_3 + 2\Psi_4 + 2\Psi_5 - 3\Psi_6 - \Psi_8 + \Psi_9 - \alpha\Psi_{10} - \alpha\Psi_{11} - \alpha\Psi_{12} + (3\alpha - \beta - 1)\Psi_{13} + (-3\alpha\delta + \beta\delta + 3\alpha - \gamma + \delta + 4)\Psi_{14}$
[1, 2, 3, 1, 2, 4, 3, 1, 2, 3]	$+\Psi_1 - 2\Psi_2 + 4\Psi_3 - 4\Psi_8 + 4\Psi_{10}$
[2, 3, 1, 2, 3, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 4\Psi_2 + 4\Psi_3 + 4\Psi_4 + 4\Psi_5 - 8\Psi_6 - 6\Psi_8 + 4\Psi_{10} + 4\Psi_{11} + 4\Psi_{12}$
[1, 3, 1, 2, 3, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 4\Psi_2 + 4\Psi_3 + 4\Psi_4 + 4\Psi_5 - 8\Psi_6 - 6\Psi_8 + 4\Psi_{10} + 4\Psi_{11} + 4\Psi_{12}$
[1, 2, 3, 1, 2, 3, 4, 3, 1, 2, 3]	$+\Psi_1 - 4\Psi_2 + 4\Psi_3 + 4\Psi_4 + 4\Psi_5 - 8\Psi_6 - 6\Psi_8 + 4\Psi_{10} + 4\Psi_{11} + 4\Psi_{12}$
[1, 2, 3, 1, 2, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 4\Psi_2 + 4\Psi_3 + 4\Psi_4 + 4\Psi_5 - 8\Psi_6 - 6\Psi_8 + 4\Psi_{10} + 4\Psi_{11} + 4\Psi_{12}$
[1, 2, 3, 1, 2, 3, 4, 3, 1, 2, 3, 4]	$+\Psi_1 - 6\Psi_2 + 8\Psi_3 + 8\Psi_4 + 8\Psi_5 - 24\Psi_6 - 16\Psi_8 - 8\Psi_9 + (8\alpha + 24)\Psi_{10} + (8\alpha + 24)\Psi_{11} + (8\alpha + 24)\Psi_{12} + (-24\alpha + 8\beta - 40)\Psi_{13} + (24\alpha\delta - 8\beta\delta - 24\alpha + 8\gamma + 40\delta - 32)\Psi_{14}$

TABLE 2. The decomposition of the Deligne-Lusztig characters on the ℓ -projective characters labelling the columns of the ℓ -decomposition matrix of G when $\ell \mid q + 1$.